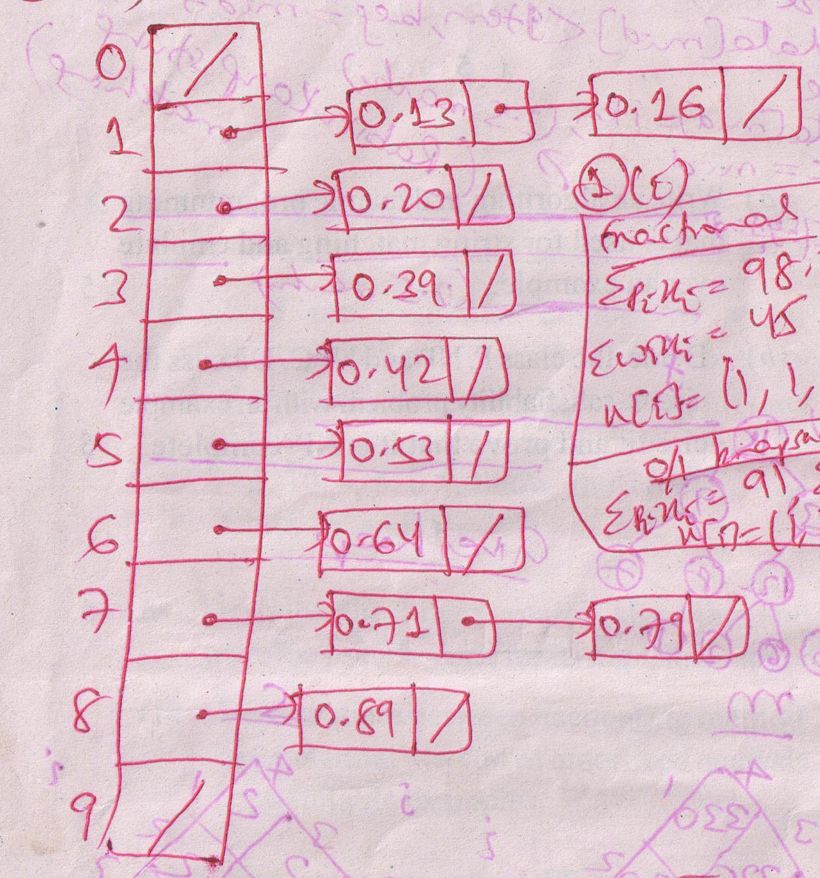
③ (b)



① (i)
fractional
$\sum p_i x_i = 98.2$
$\sum w_i x_i = 45$
$x[i] = (1, 1, 1, 1, 0.4)$

0/1 knapsack
$\sum p_i x_i = 91$ $\sum w_i x_i = 37$
$x[i] = (1, 1, 1, 1, 0)$

Operation of bucket sort

$n=5, m=45$

① (i) $P = (18, 20, 30, 17, 24)$ $w = (20, 5, 10, 15, 7)$

$P_1/w_1 = \frac{18}{20} = 0.9$ | $P_4/w_4 = 17/15 = 1.133$

$P_2/w_2 = \frac{20}{5} = 4$ | $P_5/w_5 = \frac{24}{7} = 3.42857$

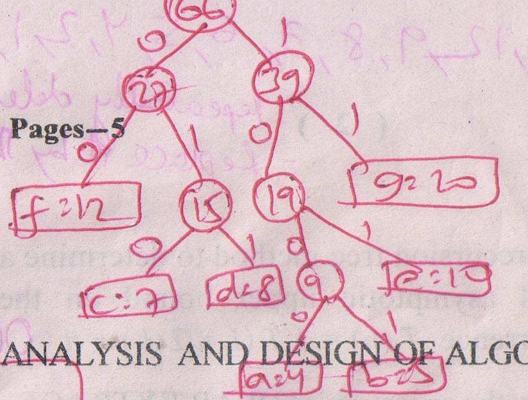
$P_3/w_3 = \frac{30}{10} = 3$ | $P_2/w_2 > P_5/w_5 > P_3/w_3 > P_4/w_4 > P_1/w_1$

$P = (20, 24, 30, 17, 18)$, $w = (5, 7, 10, 15, 20)$

$x[i] = (1, 1, 1, 1, 0.4)$

$\frac{8}{20} = 0.4$

① (b)



Total Pages—5

MCA-4

ADA

Set-1

ANALYSIS AND DESIGN OF ALGORITHM

Full Marks : 70

Time : 3 hours

Answer Q. No. 1 which is compulsory and any five from the rest

The figures in the right-hand margin indicate marks

O(1) divide
conquer 2T(n/2) times
combine O(n)

1. Answer *all* questions : **2 × 10**

(*a*) **What do you mean by divide and conquer approach ? Show the correctness of merge sort algorithm.** worst case – O(n lg n)

(*b*) **How many comparisons are necessary in the worst case to find both Maximum and Minimum of '*n*' numbers.**

comparisons

(*c*) **Show that after all edges are processed by connected-components, two vertices are in the same connected component if and only if they are in the same set.** example

(*Turn Over*)

7 (b) Max-flow min-cut theorem

If f is a flow in a flow network $G=(V, E)$ with source 's' and sink t, then the following conditions are equivalent: *

output → 15, 13, 12, 9, 8, 7, 6, 5, 4, 2, 1, 0

– Repeatedly delete the root element
– Replace it by the rightmost element

(*d*) Use recursion tree method to determine a good asymptotic upper bound on the recurrence $T(n) = 3\,T(L\; n/2\; J) + n$. $O(n \lg n)$

(*e*) Show the operation of HEAP-EXTRACT -MAX on the heap — Given A is a heap
$A = < 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 >$

(*f*) Show how quicksort performs under the assumption of balanced versus unbalanced partitioning. $O(n \lg n)$ $O(n^2)$

(*g*) If $T(n) = 8T(n/2) + n^2$, then find $O(T(n))$. case 1, $O(n^3)$

(*h*) Generate as optimal Huffman code for the following set of frequencies :
$a = 4$ $b = 5$ $c = 7$ $d = 8$ $e = 10$ $f = 12$ $g = 20$

(*i*) Find an optimal solution to the knapsack instance $n = 5$, $m = 45$, $p[1:5] = (18, 20, 30, 17, 24)$ and $w[1:5] = (20, 5, 10, 15, 7)$.

(*j*) What is the smallest value of '*n*' such that an algorithm whose running time is $100\,n^2$ runs faster than as algorithm whose running time is 2^n on the same machine ? $n = 15$

a → 1000, b → 1001, c → 010, d → 011, e → 101, f → 00, g → 11

2. (*a*) Discuss the advantages and disadvantages of Backtracking technique, giving examples. 5
saves efforts by not proceeding with those which will lead to nothing
time taking
4-queens/8 queens

(*b*) Prove that any comparison sort algorithm requires $\Omega(n \log n)$ comparisons in the worst case. 5

3. (*a*) Show that with the array representation for sorting an *n*-element heap, the leaves are the nodes indexed by
$L\; n/2\; J + 1,\; L\; n/2\; J + 2, \ldots\ldots\ldots, n.$ 5
$\lfloor n/2 \rfloor + 1, \lfloor n/2 \rfloor + 2, \ldots, n$

(*b*) Illustrate the Bucket Sort algorithm on the array of elements
$A = (0{\cdot}79, 0{\cdot}13, 0{\cdot}16, 0{\cdot}64, 0{\cdot}39, 0{\cdot}20, 0{\cdot}89, 0{\cdot}53, 0{\cdot}71, 0{\cdot}42)$. 5

4. (*a*) Why do we analyze the expected running time of a randomized algorithm and not its worst-case-running time ? 5
probability is involved

(*b*) Consider a hash table of size $m = 1000$ and a hash function, $h(k) = \lfloor m\,(kA \bmod 1) \rfloor$ for $A = (\mathrm{sqrt}(5) - 1)/2$. Compute the locations to which the keys 61, 62, 63, 64 and 65 are mapped. 5
698 316 934 552 170

$A = (\sqrt{5} - 1)/2 = 0.618$
$h(k) = \lfloor m(kA \bmod 1) \rfloor$

(7)(b) (i) f is a maximum flow in G.
(ii) The residual network G_f contains no augmenting paths.
(iii) $|f| = c(S,T)$ for some cut (S,T) of G

Bellman-Ford, Dijkstra

5. (a) Classify single source, shortest path algorithms, giving an example in each case. 5

(b) Find the minimum number of scalar multiplications and an optimal parenthesization of a matrix-chain product whose sequence of dimensions is given by < 5, 10, 3, 5, 6 >. Show the contents of tables 'm' and 's'. 5

330

$((A_1A_2)(A_3A_4))$

6. (a) Describe the binary search algorithm and write the pseudo-code for its implementation. 5

(b) Differentiate between accounting method (credit) and potential method of amortized analysis with suitable examples. 5 → potential

7. (a) Compare and contrast BFS (queue) and DFS (stack) search algorithms, discussing their suitability to problem domains. 5

capacity constraints, skew symmetry, flow conservation

(b) What are the properties of a flow in a flow network ? State the max-flow min-cut theorem. Show that it f_1 and f_2 are flows, then so is $\alpha f_1 + (1-\alpha) f_2$ for all α in the range $0 \le \alpha \le 1$. 5

prove the 3 properties of a flow

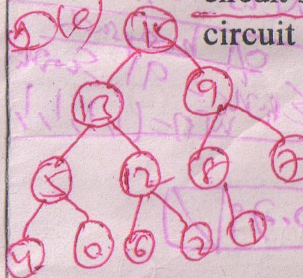
data should be in sorted order
LB=beg, UB=end, mid=int$[\frac{LB+UB}{2}]$

if data[mid] > item, end = mid − 1
else
if data[mid] < item, beg = mid + 1
else
data[mid] = item, loc = mid (2.5 marks)

(Rabin-Karp string matching)

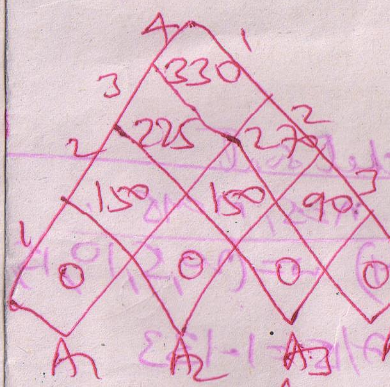
8. (a) Write an algorithm in dynamic programming that is used for string matching and emulate over an example. (2.5 marks) 5

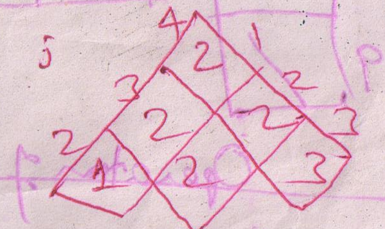
(b) Define the class P, NP and NPC. Discuss the circuit-satisfiability problem with an example circuit and prove that it is NP-complete. 5



Given heap

m





$$m[i,j] = \begin{cases} 0 & \text{if } i = j \\ \min_{i \le k < j} \{ m[i,k] + m[k+1,j] + p_{i-1}p_k p_j \} & \text{if } i < j \end{cases}$$

Optimal Parenthesization → $((A_1A_2)(A_3A_4))$